MATH 1300, Mathematical Explorations

Reasoning About Probability

Activity

- Randomness
  - Split class into groups of three. Have half of groups generate 100 random numbers with dice (or sequence of heads/tails flips with a coin) and half of groups create their own random numbers while instructor waits outside. Specify which half does what to prevent chaos. (This could also be assigned as a pre-class activity.)
  - Instructor tries to guess which sets of numbers are which and then checks results.
  - Discuss what instructor was looking for.
  - Present probabilities for chains of heads and tails in 100 flips:

# H/T	Probability	Exp # of times
4	99.97%	12
5	97.17%	6
6	80.66%	3
7	54.23%	1.5
8	31.54%	.7

  - Hand out half index cards and have each person write a random number between 1 and 100 on it
    - Tally on board and look at distribution. What would random look like?
  - Have each person predict a coin toss on their card, Heads or Tails?
    - About 80% of the time, people pick heads. When asked to predict a colored coin, they are more likely to pick whichever was said first
  - Discussion: How good are people at being random/How good are people at understanding probabilities? Possible topics/points of focus in discussion:
    - Monty Hall with pigeons
    - Gambler's fallacy: if something happens more frequently than normal during some period, it will happen less frequently in the future
    - Hector Zenil study on randomness and age
- Discussion: How can probability inform betting stratagies?
  - Martingale betting system
  - Gambler's ruin

Questions for class

- What does a random sequence look like?
- What could a computer look for in a sequence of heads and tails?

Notes

Application: iPod shuffle algorithm

Assignments

1. Read up on random number generators. Is it surprising how difficult getting random data is? Look up several uses for random numbers. Are you surprised at how important they are to our modern lives? (Expected length: 2 paragraphs.)
2. You roll two fair six-sided dice. What is the probability you get at least 6? What is the probability you roll a 3? What is the probability of getting at least six if you know one of the dice was a 3? What is the probability that you rolled a 3 if your total was at least six? (You may want to draw the whole grid out for this problem.)
3. Read the chapter from Jordan Ellenberg's "How Not to be Wrong" on Bayesian inference. (It was handed out in class.) Read about the "prosecutor's fallacy" on Wikipedia. Explain the prosecutor's fallacy from the perspective of Bayes' theorem. (Expected length: 2 paragraphs.)

References and resources

Article: The Psychology of Randomness

Article: Psychologists Know How to Predict Coin Tosses

Article: Pigeons Beat Humans Solving Monty Hall Problem

Article: Random Unpredictable Behavior

Article: When Are You Really Random? After Age 24

Lecture Notes: Random Walks and Gambler's Ruin

Wikipedia: Prosecutor's Fallacy

Follow-on activities

Monty Hall Problem